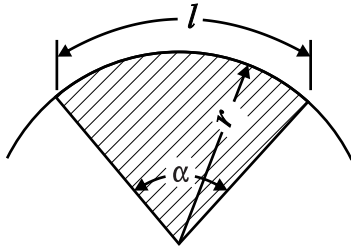


# Mensuration: finding the length ( $l$ ) of an arc

## Circular Sector



$$\text{Length of arc} = l = \frac{r \times \alpha \times 3.1416}{180} = 0.01745 r \alpha = \frac{2A}{r}$$

$$\text{Area} = A = \frac{1}{2} r l = 0.008727 \alpha r^2$$

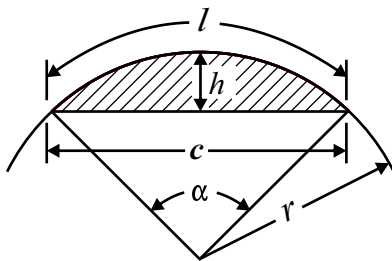
$$\text{Angle, in degrees} = \alpha = \frac{57.296 l}{r} \quad r = \frac{2A}{l} \quad \frac{57.296 l}{\alpha}$$

*Example:* The radius of a circle is 35 millimeters, and angle  $\alpha$  of a sector of the circle is 60 degrees. Find the area of the sector and the length of arc  $l$ .

$$A = 0.008727 \alpha r^2 = 0.008727 \times 60 \times 35^2 = 641.44 \text{ mm}^2 = 6.41 \text{ cm}^2$$

$$l = 0.01745 \alpha r = 0.01745 \times 35 \times 60 = 36.645 \text{ millimeters}$$

## Circular Segment



$A$  = area       $l$  = length of arc       $\alpha$  = angle, in degrees

$$c = 2\sqrt{h(2r - h)} \quad A = \frac{1}{2}[r l - c(r - h)]$$

$$r = \frac{c + 4h^2}{8h}$$

$$l = 0.01745 r \alpha$$

$$h = r - \frac{1}{2}\sqrt{4r^2 - c^2} = r[1 - \cos(\alpha/2)] \quad \alpha = \frac{57.296 l}{r}$$

*Example:* The radius  $r$  is 60 inches and the height  $h$  is 8 inches. Find the length of chord  $c$ .

$$c = 2\sqrt{h(2r - h)} = 2 \times 8 \times (2 \times 60 - 8) = 2\sqrt{896} = 2 \times 29.93 = 59.86 \text{ inches}$$

*Example:* If  $c = 16$ , and  $h = 6$  inches, what is the radius of the circle of which the segment is a part?

$$r = \frac{c + 4h^2}{8h} = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = \frac{400}{48} = 8\frac{1}{3} \text{ inches}$$

Reference: *Machinery's Handbook* 27th edition, page 66

Site Surveyor's instructions:

1. measure the distance  $c$  for the length of the signable area in on a straight line
2. measure the distance  $h$  from the straight line segment  $c$  to the maximum peak of the arc.

$c =$  \_\_\_\_\_

$h =$  \_\_\_\_\_